1

Defining the Signal

> . . . the distance of the invisible background [is] so immense that no ray from it has yet been able to reach us at all.
>
> –Edgar Allan Poe in *Eureka*, 1848

1.1 The power of light – luminosity and spectral power

The *luminosity*, L, of an object is the rate at which the object radiates away its energy (cgs units of erg s^{-1} or SI units of watts),

$$dE = L\,dt \tag{1.1}$$

This quantity has the same units as *power* and is simply the radiative power output from the object. It is an intrinsic quantity for a given object and does not depend on the observer's distance or viewing angle. If a star's luminosity is L_* at its surface, then at a distance r away, its luminosity is still L_*.

Any object that radiates, be it spherical or irregularly shaped, can be described by its luminosity. The Sun, for example, has a luminosity of $L_\odot = 3.85 \times 10^{33}$ erg s^{-1} (Table G.3), most of which is lost to space and not intercepted by the Earth (Example 1.1).

Example 1.1

Determine the fraction of the Sun's luminosity that is intercepted by the Earth. What luminosity does this correspond to?

At the distance of the Earth, the Sun's luminosity, $L_\odot$, is passing through the imaginary surface of a sphere of radius, $r_\oplus = 1$ AU. The Earth will be intercepting photons over only

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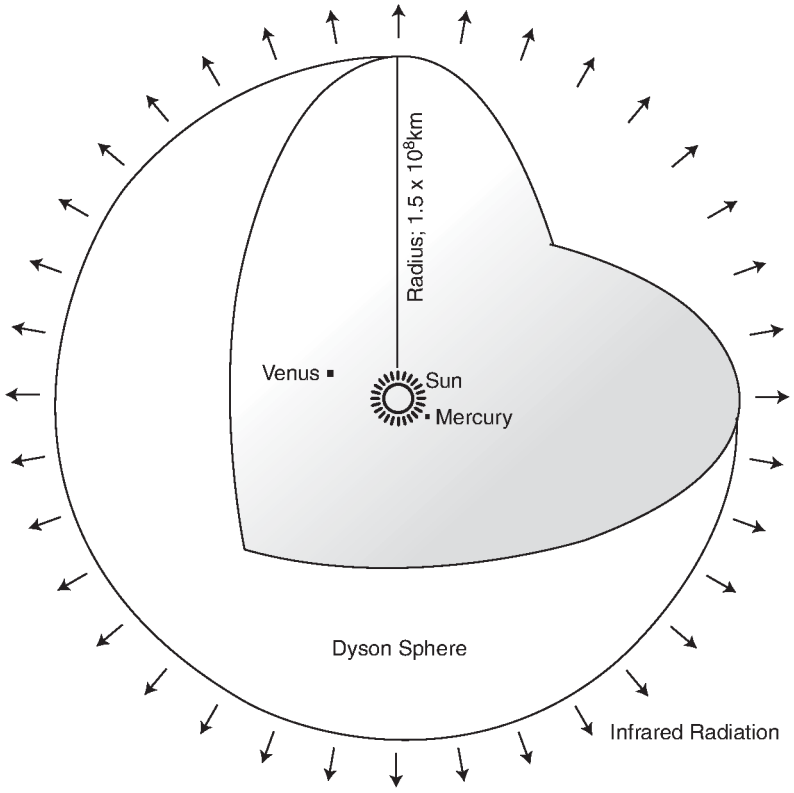

Figure 1.1. Illustration of a Dyson Sphere that could capture the entire luminous ouput from the Sun. Some have suggested that advanced civilizations, if they exist, would have discovered ways to build such spheres to harness all of the energy of their parent stars.

the cross-sectional area that is facing the Sun. This is because the Sun is so far away that incoming light rays are parallel. Thus, the fraction will be

$$\mathbf{f} = \frac{\pi {R_\oplus}^2}{4\pi r_\oplus^2} \tag{1.2}$$

where $R_\oplus$ is the radius of the Earth. Using the values of Table G.3, the fraction is $\mathbf{f} = 4.5 \times 10^{-10}$ and the intercepted luminosity is therefore $L_{\rm int} = \mathbf{f} L_\odot = 1.73 \times 10^{24}$ erg s^{-1}. A hypothetical shell around a star that would allow a civilization to intercept *all* of its luminosity is called a *Dyson Sphere* (Figure 1.1).

When one refers to the luminosity of an object, it is the *bolometric* luminosity that is understood, i.e. the luminosity over all wavebands. However, it is not possible to determine this quantity easily since observations at different wavelengths require different techniques, different kinds of telescopes and, in some wavebands, the

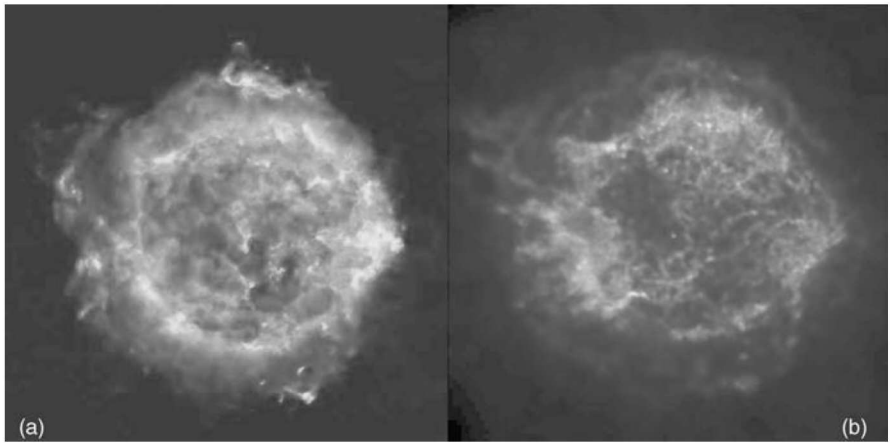

Figure 1.2. The supernova remnant, Cas A, at a distance of 3.4 kpc and with a linear diameter of $\approx$4 pc, was produced when a massive star exploded in the year AD 1680. It is currently expanding at a rate of 4000 km s^{-1} (Ref. [181]) and the proper motion (angular motion in the plane of the sky, see Sect. 7.2.1.1) of individual filaments have been observed. One side of the bipolar jet, emanating from the central object, can be seen at approximately 10 o'clock. **(a)** Radio image at λ20 cm shown in *false colour* (see Sect. 2.6) from Ref. [7]. Image courtesy of NRAO/AUI/NSF. **(b)** X-ray emission, with red, green and blue colours showing, respectively, the intensity of low, medium and high energy X-ray emission. (Reproduced courtesy of NASA/CXC/SAO) (see colour plate)

necessity of making measurements above the obscuring atmosphere of the Earth. Thus, it is common to specify the luminosity of an object for a given waveband (see Table G.6). For example, the *supernova remnant*, Cas A (Figure 1.2), has a radio luminosity (from $\nu_1 = 2 \times 10^7$ Hz to $\nu_2 = 2 \times 10^{10}$ Hz) of $L_{\text{radio}} = 3 \times 10^{35}$ erg s^{-1} (Ref. [6]) and an X-ray luminosity (from 0.3 to 10 keV) of $L_{\text{X-ray}} = 3 \times 10^{37}$ erg s^{-1} (Ref. [37]). Its bolometric luminosity is the sum of these values plus the luminosities from all other bands over which it emits. It can be seen that the radio luminosity might justifiably be neglected when computing the total power output of Cas A. Clearly, the source *spectrum* (the emission as a function of wavelength) is of some importance in understanding which wavebands, and which processes, are most important in terms of energy output. The spectrum may be represented mostly by *continuum emission* as implied here for Cas A (that is, emission that is continuous over some spectral region), or may include *spectral lines* (emission at discrete wavelengths, see Chapter 3, 5, or 9). Even very weak lines and weak continuum emission, however, can provide important clues about the processes that are occurring within an astronomical object, and must not be neglected if a full understanding of the source is to be achieved.

In the optical region of the spectrum, various *passbands* have been defined (Figure 1.3). The Sun's luminosity in V-band, for example, represents 93 per cent of its bolometric luminosity.

The *spectral luminosity* or *spectral power* is the luminosity per unit bandwidth and can be specified per unit wavelength, L_λ (cgs units of erg s^{-1} cm^{-1}) or per unit

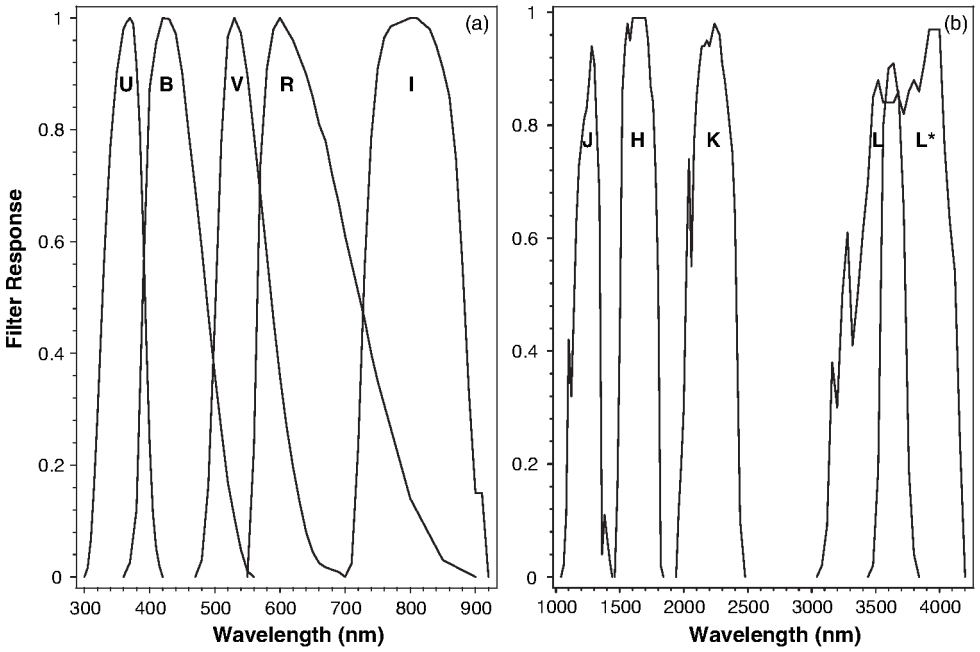

Figure 1.3. Filter bandpass responses for **(a)** the UBVRI bands (Ref. [17]) and **(b)** the JHKLL* bands (Ref. [19]). (The U and B bands correspond to UX and BX of Ref. [17].) Corresponding data can be found in Table 1.1

frequency, L_ν (erg s^{-1} Hz^{-1}),

$$dL = L_\lambda \, d\lambda = L_\nu \, d\nu \tag{1.3}$$

$$\text{so} \qquad L = \int L_\lambda \, d\lambda = \int L_\nu \, d\nu \tag{1.4}$$

Note that, since $\lambda = \frac{c}{\nu}$,

$$d\lambda = -\frac{c}{\nu^2} \, d\nu \tag{1.5}$$

so the magnitudes of L_λ and L_ν will not be equal (Prob. 1.1). The negative sign in Eq. (1.5) serves to indicate that, as wavelength increases, frequency decreases. In equations like Eq. (1.4) in which the wavelength and frequency versions of a function are related to each other, this negative is already taken into account by ensuring that the lower limit to the integral is always the lower wavelength or frequency. Note that the cgs units of L_λ (erg s^{-1} cm^{-1}) are rarely used since 1 cm of bandwidth is exceedingly large (Table G.6). Non-cgs units, such as erg s^{-1} Å^{-1} are sometimes used instead.

Luminosity is a very important quantity because it is a basic parameter of the source and is directly related to energetics. Integrated over time, it provides a measure of the energy required to make the object shine over that timescale. However, it is not a quantity that can be measured directly and must instead be derived from other measurable quantities that will shortly be described.

1.2 Light through a surface – flux and flux density

The *flux* of a source, f (erg s^{-1} cm^{-2}), is the radiative energy per unit time passing through unit area,

$$dL = f\,dA \tag{1.6}$$

As with luminosity, we can define a flux in a given waveband or we can define it per unit spectral bandwidth. For example, the *spectral flux density*, or just *flux density* (erg s^{-1} cm^{-2} Hz^{-1} or erg s^{-1} cm^{-2} cm^{-1})[1] is the flux per unit spectral bandwidth, either frequency or wavelength, respectively,

$$\begin{aligned} dL_\nu &= f_\nu\,dA \qquad & dL_\lambda &= f_\lambda\,dA \\ df &= f_\nu\,d\nu \qquad & df &= f_\lambda\,d\lambda \end{aligned} \tag{1.7}$$

A special unit for flux density, called the *Jansky* (Jy) is utilized in astronomy, most often in the infrared and radio parts of the spectrum,

$$1\,\mathrm{Jy} = 10^{-26} \qquad \mathrm{W\,m^{-2}\,Hz^{-1}} = 10^{-23} \qquad \mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}} \tag{1.8}$$

Radio sources that are greater than 1 Jy are considered to be strong sources by astronomical standards (Prob. 1.3).

The spectral response is independent of other quantities such as area or time so Eq. (1.6) and the first line of Eq. (1.7) show the same relationships except for the subscripts. To avoid repitition, then, we will now give the relationships for the bolometric quantities and it will be understood that these relationships apply to the subscripted 'per unit bandwidth' quantities as well.

The luminosity, L, of a source can be found from its flux via,

$$L = \int f\,dA = 4\pi r^2 f \tag{1.9}$$

where r is the distance from the centre of the source to the position at which the flux has been determined. The $4\pi r^2$ on the right hand side (RHS) of Eq. (1.9) is strictly

[1]The two 'cm' designations should remain separate. See the Appendix at the end of the Introduction.

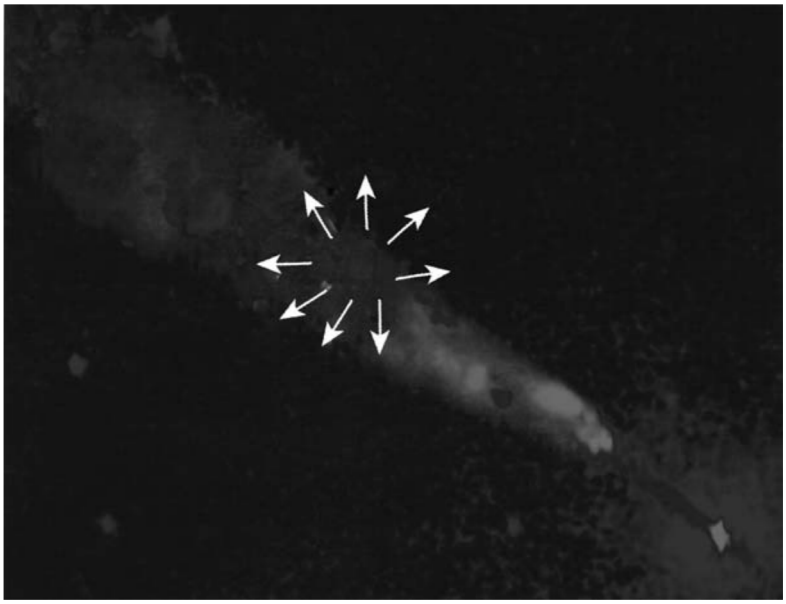

Figure 1.4. An image of the Centaurus A jet emanating from an *active galactic nucleus* (AGN) at the centre of this galaxy and at lower right of this image. Radio emission is shown in red and X-rays in blue. (Reproduced by permission of Hardcastle M.J., *et al.*, 2003 ApJ, **593**, 169.) Even though gaseous material may be moving along the jet in a highly directional fashion, the RHS of Eq. (1.9) may still be used, provided that photons generated within the jet (such as at the knot shown) escape in all directions. (see colour plate)

only true for sources in which the photons that are generated can escape in all directions, or *isotropically*. This is usually assumed to be true, even if the source itself is irregular in shape (Figure 1.4). These photons pass through the imaginary surfaces of spheres as they travel outwards. The $\frac{1}{r^2}$ fall-off of flux is just due to the geometry of a sphere (Figure 1.5.a). In principle, however, one could imagine other geometries. For example, the flux of a man-made laser beam would be constant with r if all emitted light rays are parallel and without losses (Figure 1.5.b). Light that is beamed into a narrow cone, such as may be occurring in pulsars[2] is an example of an intermediate case (Prob. 1.4).

For *stars*, we now define the *astrophysical flux*, F, to be the flux at the surface of the star,

$$L_* = 4\pi R_*^2 F = 4\pi r^2 f \qquad \Rightarrow f = \left(\frac{R_*}{r}\right)^2 F \tag{1.10}$$

where L_* is the star's luminosity and R_* is its radius.

Using values from Table G.3, astrophysical flux of the Sun is $F_\odot = 6.33\times 10^{10}\,\mathrm{erg\,s^{-1}\,cm^{-2}}$ and the *Solar Constant*, which is the flux of the Sun at the distance

[2]Pulsars are rapidly spinning *neutron stars* with strong magnetic fields that emit their radiation in beamed cones. Neutron stars typically have about the mass of the Sun in a diameter only tens of km across.

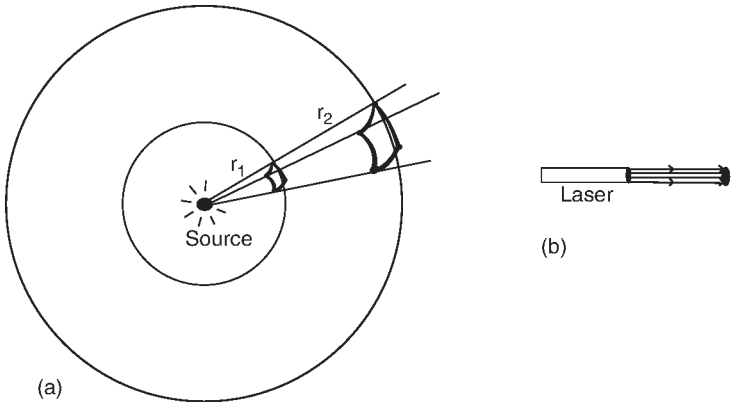

Figure 1.5. **(a)** Geometry illustrating the $\frac{1}{r^2}$ fall-off of flux with distance, r, from the source. The two spheres shown are imaginary surfaces. The same amount of energy per unit time is going through the two surface areas shown. Since the area at r_2 is greater than the one at r_1, the energy per unit time per unit cm^2 is smaller at r_2 than r_1. Since measurements are made over size scales so much smaller than astronomical distances, the detector need not be curved. **(b)** Geometry of an artificial laser. For a beam with no divergence, the flux does not change with distance.

of the Earth[3], denoted, $S_\odot$, is 1.367×10^6 erg s^{-1} cm^{-2}. The Solar Constant is of great importance since it is this flux that governs climate and life on Earth. Modern satellite data reveal that the solar 'constant' actually varies in magnitude, showing that our Sun is a variable star (Figure 1.6). Earth-bound measurements failed to detect this variation since it is quite small and corrections for the atmosphere and other effects are large in comparison (e.g. Prob. 1.5).

The flux of a source in a given waveband is a quantity that is measurable, provided corrections are made for atmospheric and telescopic responses, as required (see Sects. 2.2, 2.3). If the distance to the source is known, its luminosity can then be calculated from Eq. (1.9).

1.3 The brightness of light – intensity and specific intensity

The *intensity*, I (erg s^{-1} cm^{-2} sr^{-1}), is the radiative energy per unit time per unit solid angle passing through a unit area that is perpendicular to the direction of the emission. The *specific intensity* (erg s^{-1} cm^{-2} Hz^{-1} sr^{-1} or erg s^{-1} cm^{-2} cm^{-1} sr^{-1}) is the radiative energy per unit time per unit solid angle per unit spectral bandwidth (either frequency or wavelength, respectively) passing through unit area perpendicular to the direction of the emission. The intensity is related to the flux via,

$$df = I \cos\theta \, d\Omega \tag{1.11}$$

[3]This is taken to be above the Earth's atmosphere.

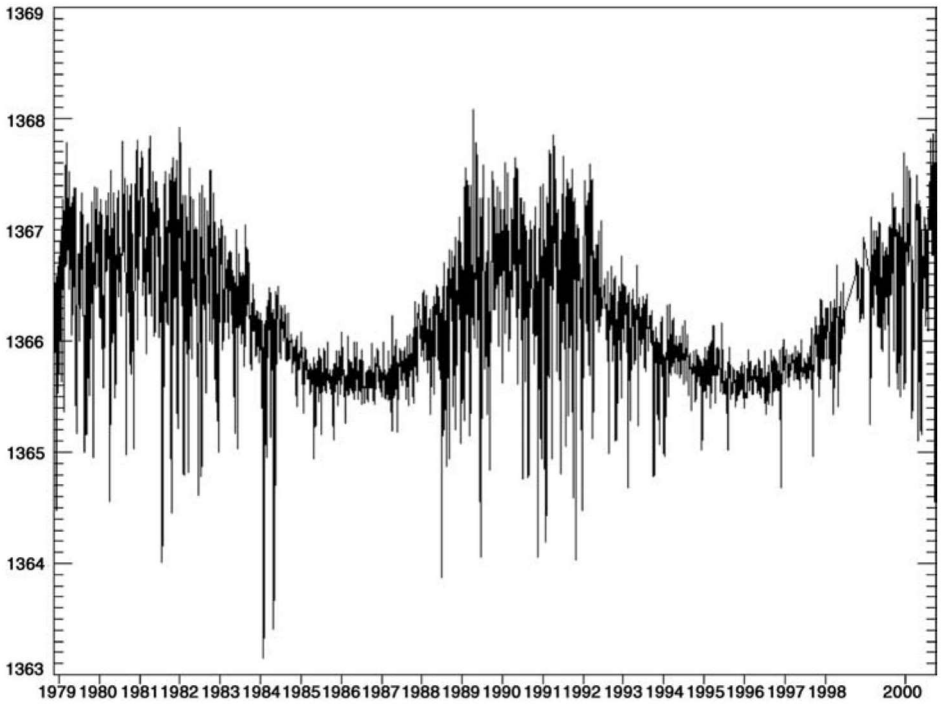

Figure 1.6. Plot of the Solar Constant (in W m^{-2}) as a function of time from satellite data. The variation follows the 11-year Sunspot cycle such that when there are more sunspots, the Sun, on average, is brighter. The peak to peak variation is less than 0.1 per cent. This plot provides definitive evidence that our Sun is a variable star. (Reproduced by permission of www.answers.com/topic/solar-variation)

As before, the same kind of relation could be written between the quantities per unit bandwidth, i.e. between the specific intensity and the flux density.

The specific intensity, I_ν, is the most basic of radiative quantities. Its formal definition is written,

$$dE = I_\nu \cos\theta \, d\nu \, d\Omega \, dA \, dt \tag{1.12}$$

Note that each elemental quantity is independent of the others so, when integrating, it doesn't matter in which order the integration is done.

The intensity isolates the emission that is within a given solid angle and at some angle from the perpendicular. The geometry is shown in Figure 1.7 for a situation in which a detector is receiving emission from a source in the sky and for a situation in which an imaginary detector is placed on the surface of a star. In the first case, the source subtends some solid angle in the sky in a direction, θ, from the zenith. The factor, cos θ accounts for the foreshortening of the detector area as emission falls on it.

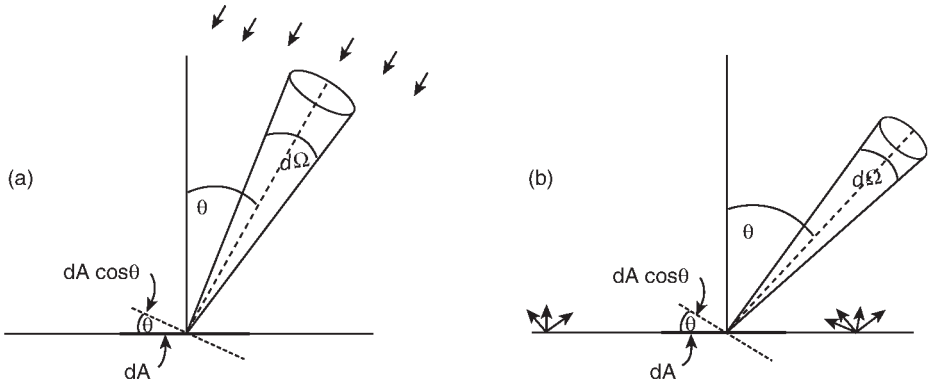

Figure 1.7. Diagrams showing intensity and its dependence on direction and solid angle, using a spherical coordinate system such as described in Appendix B. **(a)** Here *dA* would be an element of area of a detector on the Earth, the perpendicular upwards direction is towards the zenith, a source is in the sky in the direction, θ, and $d\Omega$ is an elemental solid angle on the source. The arrows show incoming rays from the *centre* of the source that flood the detector. **(b)** In this example, an imaginary detector is placed at the surface of a star. At each point on the surface, photons leave in all directions away from the surface. The intensity would be a measure of only those photons which pass through a given solid angle at a given angle, θ from the vertical

Usually, a detector would be pointed directly at the source of interest in which case $\cos\,\theta = 1$. In the second case, the coordinate system has been placed at the surface of a star. At any position on the star's surface, radiation is emitted over all directions away from the surface. The intensity refers to the emission in the direction, θ radiating *into* solid angle, $d\Omega$. Example 1.2 indicates how the intensity relates to the flux for these two examples. Figure 1.7 also helps to illustrate the generality of these quantities. One could place the coordinate system at the centre of a star, in interstellar space, or wherever we wish to determine these radiative properties of a source (Probs. 1.6, 1.7).

Example 1.2

(a) A detector pointed directly at a uniform intensity source in the sky of small solid angle, Ω, would measure a flux,

$$f = \int_{\Omega} I \cos\theta \, d\Omega \approx I\,\Omega \tag{1.13}$$

(b) The astrophysical flux at the surface of an object (e.g. a star) whose radiation is escaping freely at all angles outwards (i.e. over $2\,\pi$ sr), can be calculated by integrating

in spherical coordinates (see Appendix B),

$$F = \int I \cos\theta \, d\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I \cos\theta \sin\theta d\theta \, d\phi = \pi I \tag{1.14}$$

Figure 1.8 shows a practical example as to how one might calculate the flux of a source for a case corresponding to Example 1.2a, but for which the intensity varies with position. The intensity in a given waveband is a measurable quantity, provided a solid angle can also be measured. If a source is so small or so far away that its angular size cannot be discerned (i.e. it is *unresolved*, see Sects. 2.2.3, 2.2.4, 2.3.2), then the intensity cannot be determined. In such cases, it is the flux that is measured, as shown in Figure 1.9. All stars other than the Sun would fall into this category[4].

Specific intensity is also referred to as *brightness* which has its intuitive meaning. A faint source has a lower value of specific intensity than a bright source. Note that it is possible for a source that is faint to have a larger flux density than a source that is bright if it subtends a larger solid angle in the sky (Prob. 1.9).

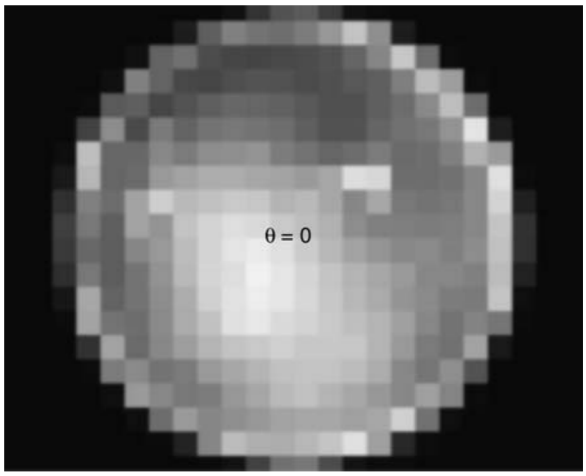

Figure 1.8. Looking directly at a hypothetical object in the sky corresponding to the situation shown in Figure 1.7.a but for $\theta = 0$ (i.e. the detector pointing directly at the source). The object subtends a total solid angle, Ω, which is small and therefore $\theta \approx 0$ at any location on the source. In this example, the object is of non-uniform brightness and Ω is split up into many small square solid angles, each of size, Ω_i and within which the intensity is I_i. Then we can approximate $f = \int I \cos\theta \, d\Omega$ using $f \approx \sum I_i \, \Omega_i$. Basically, to find the flux, we add up the individual fluxes of all elements.

[4]The exception is a few nearby stars for which special observing techniques are required.

Figure 1.9. In this case, a star has a very small angular size (left) and so, when detected in a square solid angle, Ω_p (right), which is determined by the properties of the detector, its light is 'smeared out' to fill that solid angle. In such a case, it is impossible to determine the intensity of the surface of the star. However, the flux of the star, f_*, is preserved, i.e. $f_* = I_* \Omega_* = \bar{I} \Omega_p$ (Eq. 1.13) where I_* is the true intensity of the star, Ω_* is the true solid angle subtended by the star, and $\bar{I}$ is the mean intensity in the square. Thus, for an object of angular size smaller than can be resolved by the available instruments (see Sects. 2.2.3, 2.2.4, and 2.3.2), we measure the flux (or flux density), but not intensity (or specific intensity) of the object

The intensity and specific intensity are *independent of distance* (constant with distance) in the absence of any intervening matter[5]. The easiest way to see this is via Eq. (1.13). Both f and Ω decline as $\frac{1}{r^2}$ (Eq. (1.11), Eq. (B.2), respectively) and therefore I is constant with distance. The Sun, for example, has $I_\odot = F_\odot/\pi = 2.01 \times 10^{10}$ erg s^{-1} cm^{-2} sr^{-1} as viewed from any source at which the Sun subtends a small, measurable solid angle. The constancy of I with distance is general, however, applying to large angles as well. This is a very important result, since a measurement of I allows the determination of some properties of the source without having to know its distance (e.g. Sect. 4.1).

1.4 Light from all angles – energy density and mean intensity

The *energy density*, u (erg cm^{-3}), is the radiative energy per unit volume. It describes the energy content of radiation in a unit volume of space,

$$du = \frac{dE}{dV} \tag{1.15}$$

The *specific energy density* is the energy density per unit bandwidth and, as usual, $u = \int u_\nu \, d\nu = \int u_\lambda \, d\lambda$. The energy density is related to the intensity (see Figure 1.10, Eq. 1.12) by,

$$u = \frac{1}{c} \int I \, d\Omega = \frac{4\pi}{c} J \tag{1.16}$$

[5]More accurately, I/n^2 is independent of distance along a ray path, where n is the index of refraction but the difference is negligible for our purposes.

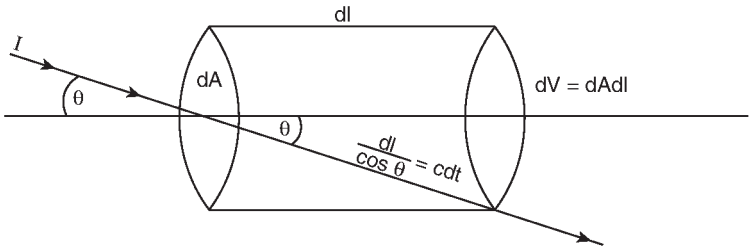

Figure 1.10. This diagram is helpful in relating the energy density (the radiative energy per unit volume) to the light intensity. An individual ray spends a time, $dt = dl/(c\,\cos\theta)$ in an infinitesimal cylindrical volume of size, $dV = dl\,dA$. Combined with Eq. (1.12), the result is Eq. (1.16).

where J is the *mean intensity*, defined by,

$$J \equiv \frac{1}{4\,\pi} \int I\,d\Omega \tag{1.17}$$

The mean intensity is therefore the intensity averaged over all directions. In an isotropic radiation field, $J = I$. In reality, radiation fields are generally not isotropic, but some are close to it or can be approximated as isotropic, for example, in the centres of stars or when considering the 2.7 K cosmic microwave background radiation (Sect. 3.1). In a non-isotropic radiation field, J is not constant with distance, even though I is. Example 1.3 provides a sample computation.

Example 1.3

Compute the mean intensity and the energy density at the distance of Mars. Assume that the only important source is the Sun.

$$\begin{aligned} J &= \frac{1}{4\,\pi} \int_0^{4\pi} I\,d\Omega \\ &= \frac{1}{4\,\pi} \int_{\Omega_\odot} I_\odot\,d\Omega \approx \frac{I_\odot\,\Omega_\odot}{4\,\pi} = \frac{I_\odot}{4\,\pi}\,\frac{\pi\,{\theta_\odot}^2}{4} = \frac{I_\odot}{16}\left(\frac{2\,R_\odot}{r_{\rm Mars}}\right)^2 \end{aligned} \tag{1.18}$$

where we have used Eq. (B.3) to express the solid angle in terms of the linear angle, and Eq. (B.1) to express the linear angle in terms of the size of the Sun and the distance of Mars. Inserting $I_\odot = 2.01 \times 10^{10}\,{\rm erg\,s^{-1}\,cm^{-2}\ sr^{-1}}$ (Sect. 1.3), $R_\odot = 6.96 \times 10^{10}$ cm, and $r_{\rm Mars} = 2.28 \times 10^{13}$ cm (Tables G.3, G.4), we find, $J = 4.7 \times 10^4\,{\rm erg\,s^{-1}\,cm^{-2}\ sr^{-1}}$. Then $u = \frac{4\pi}{c}(4.7 \times 10^4) = 2.0 \times 10^{-5}\ {\rm erg\,cm^{-3}}$.

The radiation field (u or J) in interstellar space due to randomly distributed stars (Prob. 1.10) must be computed over a solid angle of 4π steradians, given that

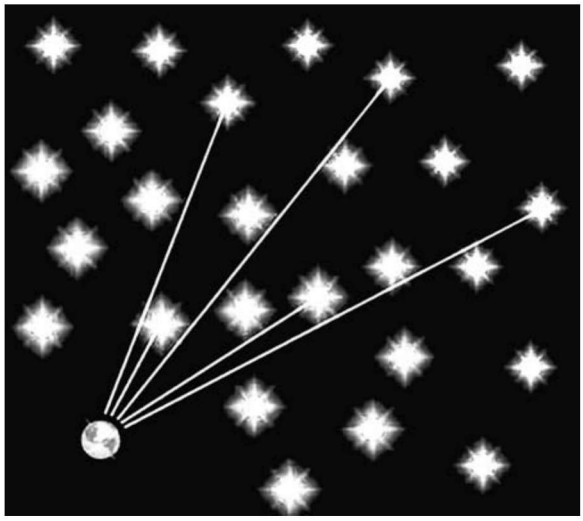

Figure 1.11. Why is the night sky dark? If the Universe is infinite and populated in all directions by stars, then eventually every sight line should intersect the surface of a star. Since I is constant with distance, the night sky should be as bright as the surface of a typical star. This is known as Olbers' Paradox, though Olbers was not the first to note this discrepancy. See Sect. 1.4.

starlight contributes from many directions in the sky. However, in this case, $J \neq I$ because there is no emission from directions between the stars. If there were so many stars that every line of sight eventually intersected the surface of a star of brightness, I_*, then $J = I_*$ and the entire sky would appear as bright as I_*. This would be true even if the stars were at great distances since I_*, being an intensity, is independent of distance. If this is the case, we would say that the stellar *covering factor* is unity.

A variant of this concept is called *Olbers' Paradox* after the German astronomer, Heinrich Wilhelm Olbers who popularized it in the 19th century. It was discussed as early as 1610, though, by the German astronomer, Johannes Kepler, and was based on the idea of an infinite starry Universe which had been propounded by the English astronomer and mathematician, Thomas Digges, around 1576. If the Universe is infinite and populated throughout with stars, then every line of sight should eventually intersect a star and the night sky as seen from Earth should be as bright as a typical stellar surface. Why, then, is the night sky dark?[6] Kepler took the simple observation of a dark night sky as an argument for the finite extent of the Universe, or at least of its stars. The modern explanation, however, lies with the

[6]The earlier form of the question was posed somewhat differently, referring to increasing numbers of stars on increasingly larger shells with distance from the Earth.

intimate relation between time and space on cosmological scales (Sect. 7.1). Since the speed of light is constant, as we look farther into space, we also look farther back in time. The Universe, though, is not infinitely old but rather had a beginning (Sect. 3.1) and the formation of stars occurred afterwards. The required number of stars for a bright night sky is $\approx 10^{60}$ and the volume needed to contain this quantity of stars implies a distance of 10^{23} light years (Ref. [74]). This means that we need to see stars at an epoch corresponding to 10^{23} years ago for the night sky to be bright. The Universe, however, is younger than this by 13 orders of magnitude (Sect. 3.1)! Thus, as we look out into space and back in time, our sight lines eventually reach an epoch prior to the formation of the first stars when the covering factor is still much less than unity. (Today, we refer to this epoch as *the dark ages*.) Remarkably, this solution was hinted at by Edgar Allan Poe in his prose-poem, *Eureka* in 1848 (see the prologue to this chapter).

1.5. How light pushes – radiation pressure

Radiation pressure is the momentum flux of radiation (the rate of momentum transfer due to photons, per unit area). It can also be thought of as the force per unit area exerted by radiation and, since force is a vector, we will treat radiation pressure in this way as well[7]. Thus, the pressure can be separated into its normal, $\mathrm{P}_\perp$, and tangential, $\mathrm{P}_\parallel$, components with respect to the surface of a wall. The normal radiation pressure will be,

$$d\mathrm{P}_\perp = \frac{d\mathrm{F}_\perp}{dA} = \frac{dp}{dt\,dA}\cos\theta = \frac{dE}{c\,dt\,dA}\cos\theta \tag{1.19}$$

where we have expressed the momentum of a photon in terms of its energy (Table I.1). Using Eq. (1.12) we obtain,

$$d\mathrm{P}_\perp = \left(\frac{1}{c}\right) I\,\cos^2\theta\,d\Omega \tag{1.20}$$

For the tangential pressure, we use the same development but take the sine of the incident angle, yielding,

$$d\mathrm{P}_\parallel = \left(\frac{1}{c}\right) I\,\cos\theta\sin\theta d\Omega \tag{1.21}$$

[7]Pressure is actually a *tensor* which is a mathematical quantity described by a matrix (a vector is a specific kind of tensor). We do not need a full mathematical treatment of pressure as a tensor, however, to appreciate the meaning of radiation pressure.